Response of an Airplane to Non-Gaussian Atmospheric Turbulence

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We study the influence of the atmospherical turbulence model on the response of a flexible airplane with nonlinear control systems. Using a new method based on filtered Poisson fields in order to build numerical simulations of non-Gaussian random fields, we show that the spectral density of the response, as well as other statistics such as level crossings or probabilistic moments, may be very different from the ones obtained when turbulence is modeled as a Gaussian process. The results presented in this work illustrate the effects of the turbulence third- and fourth-order moments.

Introduction

IN order to determine the response of an airplane to gust encounter, the power spectral density (PSD) method is often used. The spectral approach is indeed convenient to use for a rigid linear airplane since linear filtering techniques immediately give the PSD of the response and, hence, the second-order moments.[1-11] There exists also in the literature a great number of experimental data that give suitable and reliable models for the atmospheric turbulence PSD.[12,13] The higher moments of the response are, however, not correct except under the assumption that turbulence is Gaussian. For a nonlinear elastic airplane, the PSD and, thus, all of the statistics of the response cannot be obtained by this method. One way to proceed is to build simulations of turbulence, to integrate numerically the equations of the airplane motion, and to construct estimates of statistics using independent trajectories of the response. But that assumes that one is able to simulate numerically a random excitation such as turbulence. Several methods exist to generate digital simulations of Gaussian fields, but among them, the most popular is the one based on the spectral representation of Gaussian fields, which allows one to simulate homogeneous and nonhomogeneous Gaussian random fields, given their spectral measure or equivalently their autocorrelation function.[14-17] Unfortunately, as many authors have noted,[1,18-21] turbulence cannot be modeled exactly by a Gaussian random field since important characteristics such as third- or fourth-order moments, crossing rates, and extreme values are not those of a Gaussian field. Very few methods based on PSD exist in order to simulate numerically vector-valued non-Gaussian random fields, the difficulty being to restore at the same time the spectral measure and the law of the random field. In Refs. 21 and 22, the authors give a simulation method for one-dimensional non-Gaussian random processes with rational PSD and a given fourth-order moment (within some fixed bounds). In Refs. 22–25 methods are given concerning processes with given crossing rates. More recently, an iterative method was given in Ref. 26 to simulate a vector-valued non-Gaussian field given its full probabilistic law. The purpose of this article is to use a method that was developed in Refs. 27 and 28 that allows one to fix any moments of the excitation probabilistic law and, of course, the spectral measure in order to study the effect of the turbulence first moments on the response of a nonlinear flexible airplane with nonlinear gust alleviation devices. We shall see, in particular, that the control laws that were calculated assuming that the airplane was rigid and linear and assuming that turbulence was Gaussian are no longer effective when nonlinearities and non-Gaussian features are introduced. A somewhat similar study can be found in Ref. 4, where the authors looked at how the limited method given in Ref. 22 compared with a Gaussian model for a computer simulated landing aircraft. We shall also study the threshold crossing rates of the response.

Equation of the Airplane Motion

The airplane that we are considering has a delta-canard configuration with two systems of airfoils used for gust alleviation. We shall denote $\beta(t)$ and $\delta(t)$ as the angle of rotation of these airfoils. We shall take into account only the vertical component $W(m,t) = W(x,y,z,t)$ of the atmospheric turbulence. In the modal basis and in the time domain, the equation of motion is given by

$$M\ddot{Q}(t) + C\dot{Q}(t) + KQ(t) + (h*Q)(t) = (F_\beta * \beta)(t) + (F_\delta * \delta)(t) + F(t) \quad (1)$$

where $Q(t) \epsilon R^n$ represents the generalized coordinates corresponding to the n first eigenmodes; M, C, K are, respectively, the generalized mass, natural frequency, and structural damping matrices; $H(t) = (h*Q)(t)$ are the generalized aerodynamic forces; $F(t)$ is the generalized force due to atmospheric

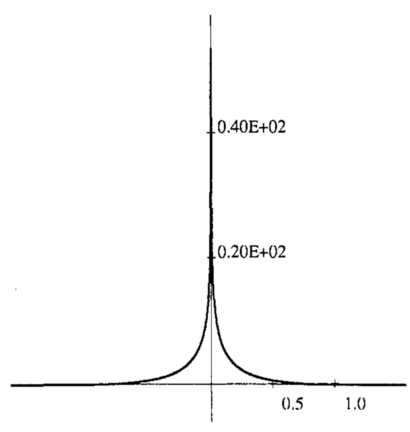

Fig. 1 Shape of a single gust (Fourier's transform of the square root of Karman's spectrum).

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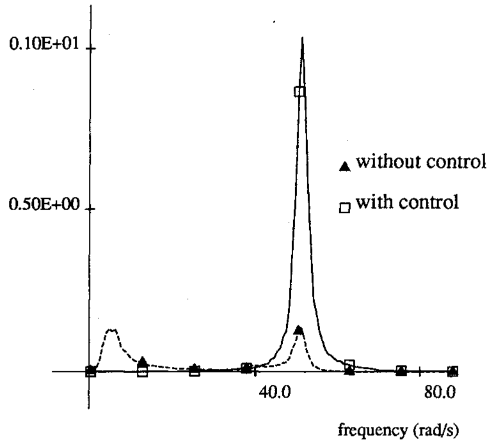

Fig. 2 Gaussian model: effect of the control on the PSD of the vertical acceleration in the cockpit.

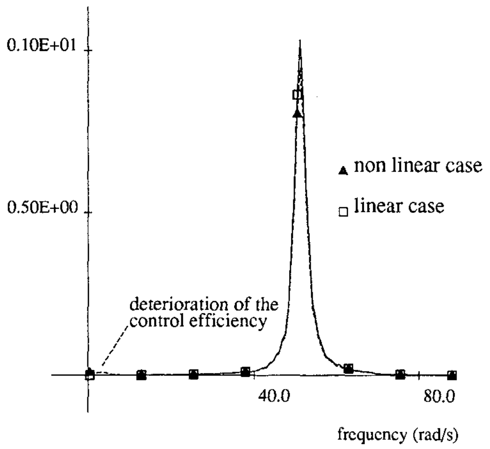

Fig. 3 Gaussian model: effect of the nonlinearities on the PSD of the vertical acceleration.

turbulence; and $\Phi_\beta = (F_\beta * \beta)(t)$ and $\Phi_\delta = (F_\delta * \delta)(t)$ are the generalized forces due to the turning of the airfoils β and δ. The Laplace transform of the functions h, F_β, and F_δ are smoothed by rational functions, and thus, the functions H, Φ_β, and Φ_δ are solutions of linear differential equations. Since in what follows we shall look solely at stationary responses, the initial conditions of these equations are arbitrary.

Equation of Airfoils Motion

The open-loop control laws for the two systems of airfoils are chosen in order to alleviate loads and motion as though the airplane behaved as a rigid linear body. Those laws are determined by using linear filtering techniques, and, thus, $\beta(t)$ and $\delta(t)$ can be seen as stationary solutions of some differential equations where the excitation or input is the turbulence. However, because of technological constraints, the motion of airfoils cannot be linear since, for instance, the angle of rotation is limited, as well as the speed of rotation itself. We consider only the last constraint on the speed:

$$|\dot{\beta}(t)| \leq \beta_0 \qquad |\dot{\delta}(t)| \leq \delta_0 \tag{2}$$

Then the differential equations describing the motion of $\beta(t)$ and $\delta(t)$ are valid only in the open domain where the speeds are strictly less than their prescribed bounds. Therefore, one must describe what happens mechanically and then mathematically when $\dot{\beta}(t)$ or $\dot{\delta}(t)$ reach their limits. Results that are based on sophisticated mathematical notions such as multivalued stochastic differential equations are given in Ref. 29 in order to model such problems of bounded random oscillations and to validate simulation techniques used to solve them numerically. Details, theoretical support, and applications can be found in Ref. 30. The constraints given by Eq. (2) will be the unique source of nonlinearities in our problem.

Generalized Force Due to Turbulence

Let V be the airplane speed. The unsteady generalized pressure field acting on the plane (S) is given by

$$P(m,t) = 1/V \int_{(S)} G(m,m',t) * W(m',t) \, \mathrm{d}m' \tag{3}$$

where $G(m,m',t)$ is the Green's function of the problem, and where the product of convolution involves the variable t. Thus, the generalized force due to turbulence and related to the displacement field $D_p(m)$ of the pth eigenmode is given by

$$F_p(t) = \int_{(S)} P(m,t) \, D_p(m) \, \mathrm{d}m, \qquad p = 1, \ldots, n \tag{4}$$

Actually, Eqs. (3) and (4) are computed numerically using a doublet lattice method on the whole plane in the frequency domain and for the nodes of a mesh $(m_i)_{i=1,\ldots,N}$ of the airplane (S). Therefore, denoting $\omega \to \hat{W}(m,\omega)$ and $\omega \to \hat{F}_p(\omega)$, the Fourier's transform of the functions $t \to W(m,t)$ and $t \to F_p(t)$,

$$\hat{F}_p(\omega) = \sum_{j=1}^{N} \sum_{k=1}^{N} D_{pj} \, G_{jk}(\omega) \, \frac{\hat{W}}{V}(m_k, \omega), \qquad p = 1, \ldots, n \tag{5}$$

where D_{pj} is the displacement of the mode p at the node m_j of the mesh, and G_{jk} is deduced from the Green's function.

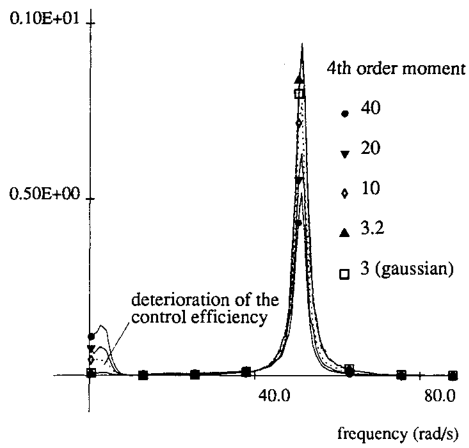

Fig. 4 Effect of the fourth-order moment of turbulence on the PSD of the vertical acceleration.

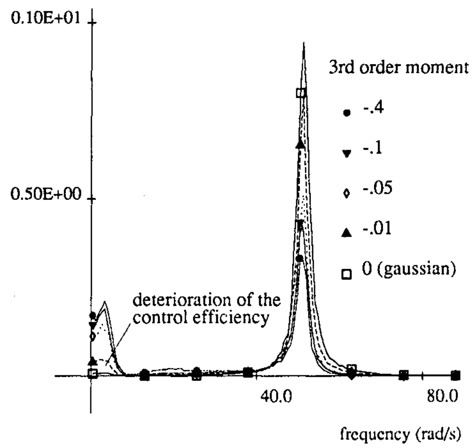

Fig. 5 Effect of the third-order moment of turbulence on the PSD of the vertical acceleration.

Use of Taylor's Hypothesis

Assuming the turbulent field is homogeneous and frozen, we can transform the expression of the Fourier's transform $\hat{W}(\boldsymbol{m},\omega) = \hat{W}(x,y,z,\omega)$. As a matter of fact, we have $W(x,y,z,t) = W(x - Vt,y,z,0)$. Dropping the time entry, this last quantity will be written in the following $W(x - Vt,y,z)$. Therefore, taking the Fourier's transform for the variable t gives

$$\int_{-\infty}^{+\infty} e^{i\omega t} W(x-Vt,y,z)\,\mathrm{d}t = 1/V e^{i\omega/Vx} \int_{-\infty}^{+\infty} e^{i\omega/Vx'} \times W(x',y,z)\,\mathrm{d}x' = 1/V e^{i\omega/Vx}\hat{W}(\omega/V,y,z) = 1/V e^{ik_1x}\hat{W}(k_1,y,z) \tag{6}$$

where $k_1 = \omega/V$ is the first component of the wave vector $\boldsymbol{k} = (k_1,k_2,k_3)$.

Non-Gaussian Model for Turbulence

The model that we use in order to generate digital simulations of non-Gaussian fields is based on the notion of filtered Poisson processes. One-dimensional filtered Poisson processes have been studied in engineering problems, such as vehicle loads on bridges, by several authors.[31-35] In Ref. 28, we generalize one-dimensional process results to vector-valued random fields. We show also that it is possible to fix the first moments of the law. In what follows, we recall the definition and the main results.

Let $(\tau_{j,n})_{j=1,2,3}$ be three independent sequences of real valued random variables modeling the occurrence times of three independent Poisson processes with parameters γ_j, $j = 1,2,3$; let $(Y_n)_{n\epsilon N}$ be a sequence of independent random variables that have the same law as a given random variable Y; and let $f:(x,y,z)\rightarrow f(x,y,z)$ be a real valued function that checks some properties. Let $w(x,y,z)$ be the random field defined by

$$w(x,y,z) = \sum_{l,m,n\epsilon N} Y_l\, f(x-\tau_{1,l},\, y-\tau_{2,m},\, z-\tau_{3,n}) \tag{7}$$

then $w(x,y,z)$ converges in law toward a homogeneous random field $W(x,y,z)$ when x, y, and z converge toward infinity. The spectral measure density of W is then given by

$$S_W(k_1,k_2,k_3) = \gamma_1\gamma_2\gamma_3/(2\pi)^3 E(Y^2)\,|(\mathfrak{F}f)(k_1,k_2,k_3)|^2 \tag{8}$$

where

$$(\mathfrak{F}f)(k_1,k_2,k_3) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-i(k_1x+k_2y+k_3z)} \times f(x,y,z)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \tag{9}$$

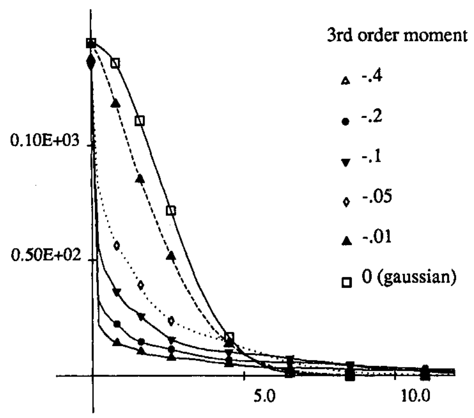

Fig. 6 Mean number of level crossings: effect of the fourth-order moment of turbulence.

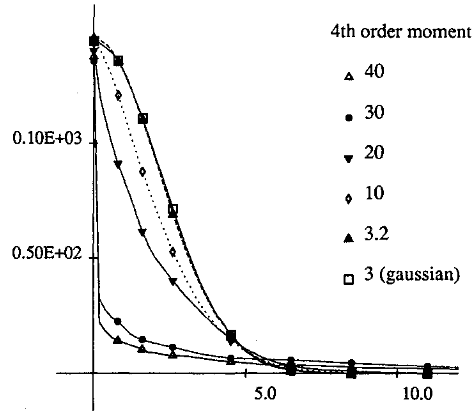

Fig. 7 Mean number of level crossings: effect of the third-order moment of turbulence.

Reciprocally, if the spectral density $S_W(k_1,k_2,k_3)$ is given and if we consider the function f given by

$$f(x,y,z) = \frac{[\gamma_1\gamma_2\gamma_3E(Y^2)]^{1/2}}{(2\pi)^3}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{i(k_1x+k_2y+k_3z)} \times [S_W(k_1,k_2,k_3)]^{1/2}\,\mathrm{d}k_1\,\mathrm{d}k_2\,\mathrm{d}k_3 \tag{10}$$

then the random field w defined by Eq. (7) will converge toward a homogeneous field with the same spectral density S_W. Since we know how to fix the spectral density, let us look now at the law of the field. Between the cumulants of the law of $W(x,y,z)$ and the moments of the law of the random variable Y exist some simple relations. For instance, the second- and fourth-order cumulants κ_2 and κ_4 of W are given by

$$\kappa_2 = \gamma_1\gamma_2\gamma_3E(Y^2)\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f^2(x,y,z)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \tag{11a}$$

$$\kappa_4 = \gamma_1\gamma_2\gamma_3E(Y^4)\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f^4(x,y,z)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \tag{11b}$$

Therefore, knowing the first L cumulants of the law of W we can deduce L real numbers, which, if they satisfy the moments condition,[36] are effectively the first L moments of a random variable Y, which, once replaced in Eqs. (11), will give back the wanted cumulants for W. If, for instance, W is Gaussian, the L numbers obtained earlier do not check the moment condition and, therefore, this method cannot be used to simulate Gaussian fields. Numerical methods are given in Ref. 36 to simulate random variables with given moments. The arrival times $\tau_{j,n}$ are also easy to simulate. The parameters γ_j are related to the sampling steps Δx, Δy, Δz of the random field:

$$\gamma_1 = 1/\Delta x, \qquad \gamma_2 = 1/\Delta y, \qquad \gamma_3 = 1/\Delta z$$

We can now give the expression used in order to construct a simulated trajectory of the generalized force $F(t) = [F_1(t), \ldots, F_n(t)]$:

$$\hat{F}_p(\omega) = \sum_{j=1}^{N}\sum_{k=1}^{N} D_{pj}\, G_{jk}(\omega) \sum_{l,m,n} \frac{Y_l}{V^2} e^{i\omega/Vx_k} e^{i\omega/V\tau_{1,l}} \times \hat{f}(\omega/V, y_k-\tau_{2,m}, z_k-\tau_{3,n}) \tag{12}$$

where

$$\hat{f}(k_1,y,z) = \int_{-\infty}^{+\infty} e^{-ik_1x} f(x,y,z)\,\mathrm{d}x$$

Taking the inverse Fourier's transform of $\hat{F}_p(\omega)$ gives the simulated trajectory $t \rightarrow F_p(t)$ of the pth component of $F(t)$. Having a trajectory of the excitation, we can then solve the differential Eq. (1) using a step-by-step integration method in order to get the response of the airplane.

Application

We consider in this section the vertical turbulence modeled by a one-dimensional stationary process whose spectral density is given by Kármán's spectrum:

$$S(\omega) = \frac{\sigma^2 L}{2\pi V} \frac{1 + 8/3\ (1.339\ L\omega/V)^2}{[1 + (1.339\ L\omega/V)^2]^{11/6}} \tag{13}$$

where σ is the turbulence standard deviation, L the turbulence length scale, and V the aircraft speed. We construct the following filtered Poisson process:

$$w(t) = \sum_{n=0}^{+\infty} Y_n f(t - \tau_n) \tag{14}$$

where f is given by

$$f(t) = \sqrt{\tfrac{1}{2}\pi}[\gamma E(Y^2)]^{-\frac{1}{2}} \int_{-\infty}^{+\infty} e^{i\omega t} \sqrt{S(\omega)}\ \mathrm{d}\omega \tag{15}$$

Figure 1 shows the graph of the function f which can be interpreted as giving the shape of a single gust. The random variables $(\tau_n - \tau_{n-1})$ are independent and have an exponential law with parameter γ. Thus, the occurrence times (τ_n) can be simulated using the following relation:

$$\tau_n = -1/\gamma \sum_{j=1}^{n} \log(1 - U_j) \tag{16}$$

where U_j are independent random variables with a uniform law over [0,1] and $\gamma = 1/\Delta t$, Δt being the time step used both to sample the random process and to discretize the various differential equations.

For this particular application, we study the effect of the third- and fourth-order moments of the turbulence on the vertical acceleration $\ddot{Z}(t)$ in the cockpit, and we compare it with the result obtained when the turbulence is assumed to be Gaussian. Using the different tools described earlier, we get as a result a discretized trajectory of the vertical acceleration: $\ddot{Z}(\Delta t)$, $\ddot{Z}(2\Delta t), \ldots, \ddot{Z},(N\Delta t)$. We construct a high number $\mathfrak{N}$ of such (independent) trajectories, and we build the estimate of the spectral density $S_{\ddot{z}}(\omega)$ of the process $\ddot{Z}(t)$. Then we perform a parametric study, changing the values of the third- and fourth-order moments of turbulence. Figure 2 represents the comparison of the spectral density obtained when the plane is controlled and not controlled, assuming that the turbulence is Gaussian and the dynamical structure is linear. We see that the effect of the gust alleviation system suppresses the response of the plane in the low frequencies (rigid body) domain but increases the response in the flexible modes domain. That is due to the specific control laws chosen; they may not be the most adequate. The purpose of this work is not to find the optimal control laws but to show, for given laws, the effect of the excitation probabilistic model on the response. Figure 3 represents the comparison between the acceleration PSD for a linear and nonlinear model of the airplane, turbulence being always Gaussian. We see now, that in the case where nonlinearities are introduced, the control laws that were determined assuming that the plane is linear are less efficient in the nonlinear case since the response has some energy in the low frequencies range.

Now, changing the values of the third- and fourth-order moments of turbulence always with the given spectral density [Eq. (13)], we can estimate the various response PSD obtained. Figure 4 shows the comparison between the results when the fourth-order moment value is increased; Fig. 5 shows the same results when only the third-order moment is modified. We see in both cases that the spectral density of the response is modified, which means that second-order techniques cannot be used in nonlinear stochastic systems, and we see also that the control laws become less and less efficient as the excitation gets further from a Gaussian process. This last fact means that, in order to determine good control laws, one has to take into account, for nonlinear systems, the non-Gaussian characteristics of turbulence. Besides second-order results, we have also looked at statistical results on the response trajectories; we estimate the crossing rates of the vertical acceleration for given thresholds, since those results are important for failure or reliability problems. Figures 6 and 7 show, respectively, the influence of the fourth- and third-order moments of turbulence on those crossing rates for different values of thresholds. We see, in particular, that the mean number of crossings are more important in the non-Gaussian cases for the high values of thresholds. The curve obtained when the excitation is Gaussian, which is given by a classic formula, is very often used by engineers even when the excitation is not Gaussian and the dynamical system nonlinear, which may lead to important errors.

Conclusions

In order to study the response of a nonlinear dynamical system to a stationary random excitation with a given PSD as, for instance, the response of a nonlinear flexible aircraft to atmospherical turbulence, we have shown that it is necessary to take into account the probabilistic distribution of the excitation. Making a Gaussian assumption can lead to important errors not only in the probabilistic distribution of the response itself but also in its PSD. However, the complete law is usually unknown; one has access only to some partial information, for instance, the values of the first moments of the excitation law. When this is the case, a method of simulation exists that fulfills the following requirements: to give back both the given PSD and those first moments. We have applied this method to study the problem of a controlled airplane flying in turbulence. The results obtained highlight the necessity of using realistic models for excitation and lead to the identification problem of probabilistic characteristics of random phenomena. As far as atmospherical turbulence is concerned, not much attention has been given in the past to the determination of its probabilistic moments, although such quantities are easy to estimate on experimental measures. In this paper we limited the study to the effect of the third- and fourth-order moments. Higher-order moments could have been introduced, as well, without increasing the total CPU time of the numerical calculations.

References

[1]Taylor, J., *Manual on Aircraft Loads,* AGARDograph 83, Pergamon, New York, 1965.

[2]Etkin, B., "Turbulent Wind and Its Effect on Flight," *Journal of Aircraft,* Vol. 18, No. 5, May 1981, pp. 327–345.

[3]Frost, W., and Wang, S. T., "Atmospheric Turbulence. Simulation Techniques with Application to Flight Analysis," NASA CR 3309, 1980.

[4]Barnes, T. D., "Current and Proposed Gust Criteria and Analysis Methods. An FAA Overview," AGARD Rept. 734, Oct. 1987.

[5]Moon, R. N., "A Summary of Methods for Establishing Airframe Design Loads from Continuous Gust Design Criteria," AGARD Rept. 734, Oct. 1987.

[6]Glaser, J. J., "Data Requirements on Turbulence in the Earth's Atmospheric Shear Layer for STOL Design Criteria," AGARD CP-140, May 1973.

[7]Gerlach, O. H., Van der Moesdijk, G. A. J., and Van der Vaart, J. C., "Progress in the Mathematical Modeling of Flight in Turbulence," AGARD CP-140, May 1973.

[8]Grisval, J. P., Meurzec, J. L., and Poirion, F., "Numerical Method for the Response of an Aircraft to the Atmospheric Turbulence. Application to the NORD 260 Airplane," *Proceedings of the European Forum on Aeroelasticity and Structural Dynamics,* DGLR, Bonn, Germany, April 1989.

[9]Campbell, W., "Monte Carlo Turbulence Simulation Using Rational Approximations to Von Kármán Spectrum," *AIAA Journal,* Vol.

24, No. 1, 1986, pp. 62–66.
[10]Tatom, F., and Smith, S., "Simulation of Atmospheric Turbulent Gusts and Gust Gradients," *Journal of Aircraft*, Vol. 19, No. 4, 1982, pp. 264–271.
[11]Houbolt, J. C., "Gust Design Procedures Based on Power Spectral Techniques," Air Force Flight Dynamics Lab., Wright-Patterson AFB, OH, AFFDL-TR-67-74, Aug. 1967.
[12]Kaimal, J. L., Wyngaard, J. C., Izumi, Y., and Cote, O. C., "Spectral Characteristics of Surface Layer Turbulence," *Quarterly Journal of the Royal Meteorological Society,* Vol. 98, 1972, pp. 563–589.
[13]Teunissen, H. W., "Structure of Mean Winds and Turbulence in the Planetary Boundary Layer over Rural Terrains," *Boundary Layer Meteorology,* Vol. 19, 1980, pp. 187–2201.
[14]Shinozuka, M., "Simulations of Multivariate and Multidimensional Random Processes," *Journal of the Acoustical Society of America,* Vol. 49, No. 1, Pt. 2, 1971, pp. 357–367.
[15]Shinozuka, M., and Wen, Y. K., "Monte Carlo Solution of Nonlinear Vibrations," *AIAA Journal,* Vol. 10, No. 1, 1972, pp. 37–40.
[16]Poirion, F., and Soize, C., "Numerical Simulation of Homogeneous and Inhomogeneous Gaussian Stochastic Vector Fields," *Recherche Aérospatiale,* No. 1, 1989, pp. 42–61.
[17]Vacaitis, R., Jan, C.-M., and Shinozuka, M., "Nonlinear Panel Response from a Turbulent Boundary Layer," *AIAA Journal*, Vol. 10, No. 7, 1972, pp. 895–899.
[18]Jones, J. G., "A Theory of Extreme Gust Loads on an Aircraft Based on the Representation of the Atmosphere as a Self-Similar Intermittent Random Process," Royal Aircraft Establishment, Farnborough, England, UK, TR-68030, 1968.
[19]Jones, J. G., "Statistical-Discrete-Gust Method for Predicting Aircraft Loads and Dynamic Response," *Journal of Aircraft*, Vol. 26, No. 4, 1988, pp. 382–392.
[20]Trevino, G., "Turbulence for Flight Simulation," *Journal of Aircraft*, Vol. 23, No. 4, 1986, pp. 348,349.
[21]Mark, W., "Characterization of Non-Gaussian Atmospheric Turbulence for Prediction of Aircraft Response Statistics," NASA CR-2745.
[22]Reeves, P. M., Joppa, R. G., and Ganzer, V. M., "A Non-Gaussian Model of Continuous Atmospheric Turbulence for Use in Aircraft Design," NASA CR-2639, Jan. 1976.
[23]Shinozuka, M., and Tan, R. Y., "Probabilistic Load Combinations and Crossing Rates," *Proceedings of the Symposium on Probabilistic Methods in Structural Engineering,* edited by M. Shinozuka and J. T. P. Yao, American Society of Civil Engineers, New York, 1981, pp. 229–250.
[24]Grigoriu, M., "Crossing of Non-Gaussian Translation Process," *Journal of the Engineering Mechanics Division,* Vol. 110, No. 4, 1984, pp. 610–620.
[25]Wilkins, D. J., Wolff, R. V., Shinozuka, M., and Cox, E. F., "Realism in Fatigue Testing: The Effect of Flight-by-Flight Thermal and Random Load Histories on Composite Bonded Joints," American Society of Testing and Materials, Philadelphia, STP-569, 1975, pp. 307–322.
[26]Shinozuka, M., and Yamazaki, M., "Digital Generation of Non-Gaussian Stochastic Fields," *Journal of the Engineering Mechanics Division,* Vol. 114, No. 7, 1988, pp. 1183–1197.
[27]Poirion, F., "Modèle de Simulation Numérique avec Contrôle des Statistiques sur les Trajectoires," ONERA, Châtillon, France, 5168RH081R, March 1989.
[28]Poirion, F., "Numerical Simulation of Homogeneous Non-Gaussian Random Fields," *Journal of Sound and Vibration* (submitted for publication).
[29]Poirion, F., "Bounded Random Oscillations: Model and Numerical Resolution for an Airfoil," *Proceedings of the IUTAM Symposium on Nonlinear Stochastic Dynamic Engineering Systems,* edited by F. Ziegler and G. I. Schueller, Springer-Verlag, Berlin, 1987, pp. 243–2051.
[30]Krée, P., and Soize, C., *Mathematics of Random Phenomena,* Reidel, Dordrecht, The Netherlands, 1986.
[31]Liu, S. C., "Dynamics of Correlated Random Pulse Trains," *Journal of the Engineering Mechanics Division,* Vol. 96, No. 5, 1970, pp. 455–471.
[32]Raccicot, R., and Moses, F., "Filtered Poisson Process for Random Vibration Problems," *Journal of the Engineering Mechanics Division,* Vol. 98, 1972, pp. 159–175.
[33]Tung, C. C., "Random Response of Highway Bridges to Vehicle Loads," *Journal of the Engineering Mechanics Division,* Vol. 93, No. 5, 1967, pp. 79–94.
[34]Lin, Y. K., "On Nonstationary Shot Noise," *Journal of the Acoustical Society of America,* Vol. 36, 1964, pp. 82–84.
[35]Shinozuka, M., and Sato, Y., "Simulation of Nonstationary Random Process," *Journal of the Engineering Mechanics Division,* Vol. 93, No. 1, New York, 1967, pp. 11–40.
[36]Devroye, L., *Non-Uniform Random Variate Generation,* Springer-Verlag, New York, 1986.